MATHEMATICS SPECIALIST

MAWA Year 12 Examination 2018

Calculator-free

Marking Key

The release date for this exam and marking scheme is

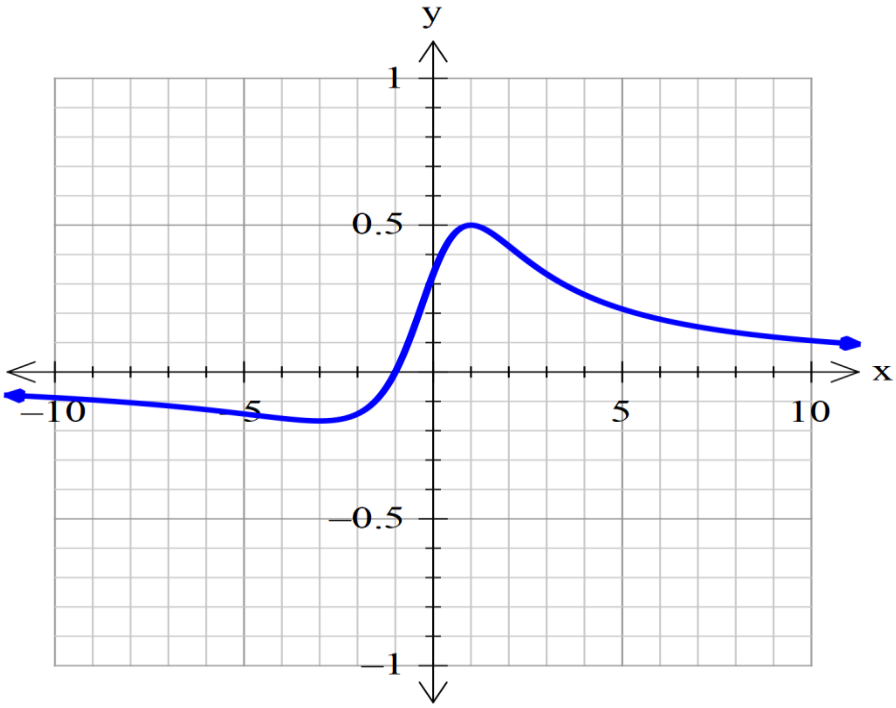
- **the end of week 8 of term 2, 2018**

Question 1 **(4 marks)**

Solution
Now $\overrightarrow{PQ} = (-1, a-6, 5)$ and $\overrightarrow{PR} = (b-2, -1, 3)$. (note that $\overrightarrow{QR} = (b-1, 5-a, -2)$) As PQR is a straight line, these two vectors are parallel and are linearly related. Inspection of the **k** components show that $3\ \overrightarrow{PQ} = 5\ \overrightarrow{PR}$ Comparison of the **i** components gives $-3 = 5(b-2) \Rightarrow 5b = 7 \Rightarrow b = \frac{7}{5}$. Comparison of the **j** components gives $3(a-6) = -5 \Rightarrow 3a = 13 \Rightarrow a = \frac{13}{3}$

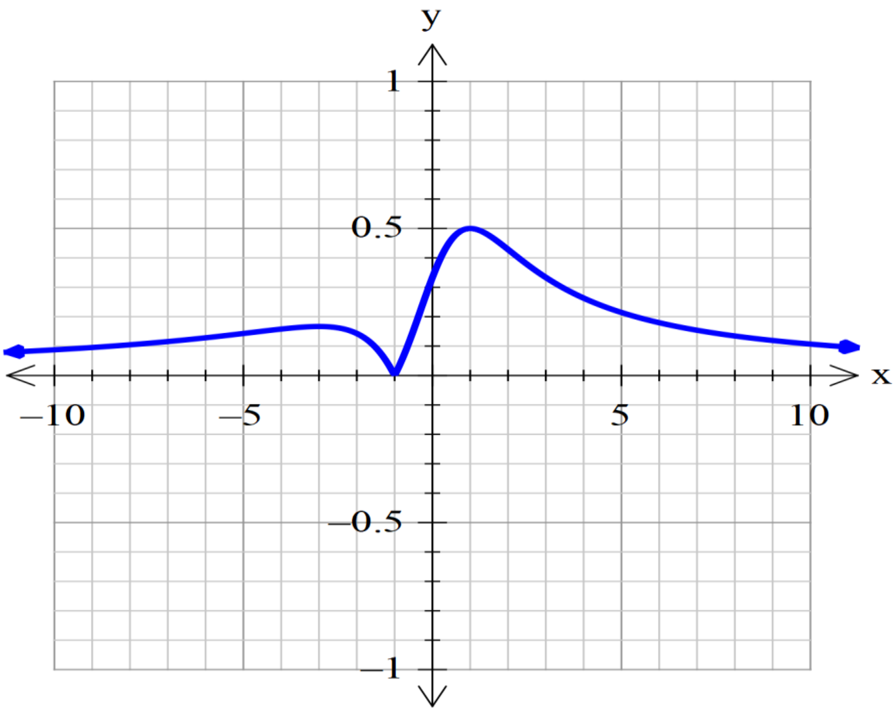
Mathematical behaviours	Marks
• calculates correctly two of the vectors $\overrightarrow{PQ}, \overrightarrow{PR}$ and $\overrightarrow{QR}$	1
• uses the k components of the two vectors to establish the linear relationship	1
• compares the first components to determine the value of b	1
• compares the second components to determine the value of a	1

Question 2(a) **(9 marks)**

Solution
As $x \to \pm\infty$ clearly have that $f(x) \to 0$. Also $$f'(x) = \frac{(x^2+3).1-(x+1)(2x)}{(x^2+3)^2} = \frac{3-2x-x^2}{(x^2+3)^2} = \frac{(3+x)(1-x)}{(x^2+3)^2}.$$ Turning points at $x = 1, x = -3$. Also $f(1) = 1/2$ and $f(-3) = -1/6$. Function is zero at $x = -1$ and $f(0) = 1/3$ 

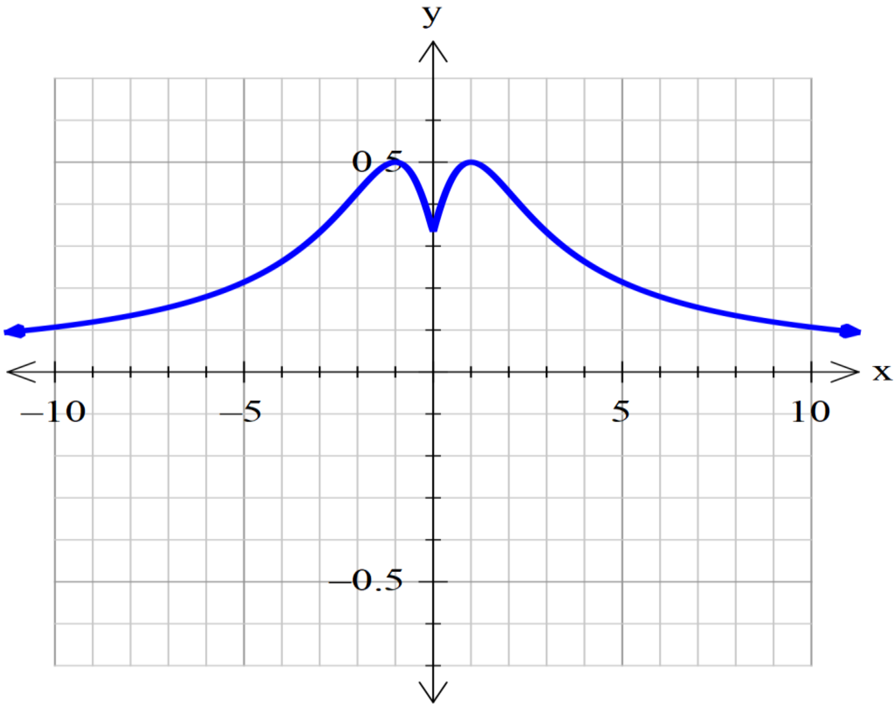
Mathematical behaviours	Marks
• identifies correct behaviour for large values of x	1
• differentiates using the quotient rule	2
• identifies the co-ordinates of one turning point…..	1
• ……and of the other turning point	1
• identifies where function crosses the co-ordinate axes	1
• draws a neat sketch with a function with properly identified max/min…	1
• …….correct large x behaviour…..	1
……..and correct overall shape	1

Question 2b(i) **(3 marks)**

• Solution
Denominator of the function is positive. The part of the curve in (a) that was below the x-axis is now reflected in the x-axis 

Mathematical behaviours	Marks
• draws a sketch of the required function with evidence of recognising that $g(x) = \|f(x)\|$	1
• the reflection of the part of the function lying in $x < -1$ and	1
• a clear discontinuity in the slope of the graph either side of $x = -1$	1

Question 2b(ii) **(3 marks)**

Solution
Function is now an even function since the denominator is clearly even. The part of the curve in (a) that was to the right of the y-axis is now reflected in the y axis 

Mathematical behaviours	Marks
• indicates an appreciation that the modified function $h(x) = f(\|x\|)$	1
• draws a neat sketch of the required function….	1
..with evidence of the reflection of the part of the function lying in $x < 0$	1

Question 3 (a) **(2 marks)**

Solution	
$z = \frac{1+i\sqrt{3}}{1+i} \times \frac{1-i}{1-i} = \frac{1+\sqrt{3}+i(\sqrt{3}-1)}{2}$ Hence $\operatorname{Re} z = \frac{1+\sqrt{3}}{2}, \operatorname{Im} z = \frac{\sqrt{3}-1}{2}$	
Mathematical behaviours	**Marks**
• gives correct value for $\operatorname{Re} z$ • gives correct value for $\operatorname{Im} z$	1 1

Question 3 (b) **(3 marks)**

Solution	
$\arg(1+i\sqrt{3}) = \frac{\pi}{3}$ and $\arg(1+i) = \frac{\pi}{4}$ Hence $\arg z = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$ Therefore $\tan\frac{\pi}{12} = \frac{\operatorname{Im}(z)}{\operatorname{Re}(z)} = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{3+1-2\sqrt{3}}{2} = 2-\sqrt{3}$	
Mathematical behaviours	**Marks**
• states correct value of $\arg(1+i\sqrt{3})$ and $\arg(1+i)$ • deduces correct value of $\arg z$ • gives correct value of $\tan(\pi/12)$	1 1 1

Question 4 (a) **(2 marks)**

Solution	
The direction vector $\overrightarrow{AB} = (-7, 3, 0)$ Thus required vector equation is **r** = 4 **i** – 2 **j** + **k** $+\lambda$ $(-7$ **i** + 3 **j**)	
Mathematical behaviours	Marks
• calculates correctly the direction vector • writes down an acceptable form of the vector line	1 1

Question 4 (b) **(2 marks)**

Solution	
The point on the line with the first component 18 corresponds to $4-7\lambda=18 \Rightarrow \lambda=-2$ Then **r** = 18 **i** -8 **j** + **k** so that we have m=-8 and n=1.	
Mathematical behaviours	Marks
• uses the given point to infer the value of λ • hence deduces the correct values of m and n	1 1

Question 4 (c) **(2 marks)**

Solution	
If $x=4-7\lambda,\ y=3\lambda-2 \Rightarrow \lambda=\dfrac{4-x}{7}=\dfrac{y+2}{3}$ Now the z-coordinate is constant so that the Cartesian equation is $\dfrac{4-x}{7}=\dfrac{y+2}{3};\ z=1$	
Mathematical behaviours	Marks
• obtains the correct relationship between x and y • deduces the correct equation including the property that z is constant.	1 1

Question 4 (d) **(1 mark)**

Solution	
As the z co-ordinate is constant, the line lies in the plane $z=1$ parallel to the xy-plane	
Mathematical behaviours	Marks
• characterises the line as being parallel to the xy-plane	1

Question 5 (a) **(2 marks)**

Solution	
$\lvert z\rvert^2=\left(\frac{-\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2=\frac{3}{4}+\frac{1}{4}=1$. Hence $\lvert z\rvert=1$ Also the argument of z lies in the second quadrant with $\arg z=\tan^{-1}\left(\frac{1}{-\sqrt{3}}\right)=\pi-\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)=\pi-\frac{\pi}{6}=\frac{5\pi}{6}$	
Mathematical behaviours	**Marks**
• states the correct value for $\lvert z\rvert$ • gives the correct value for $\arg z$	1 1

Question 5 (b) **(3 marks)**

Solution	
From part (a) we know that $z=$ cis$\left(\frac{5\pi}{6}\right)$ and so $z^{12}=$ cis$(10\pi)=1$ Now $2018=168\times12+2$ and so $z^{2018}=z^2=$ cis$\left(\frac{5\pi}{3}\right)=\cos\left(\frac{\pi}{3}\right)-i\sin\left(\frac{\pi}{3}\right)=\frac{1}{2}(1-i\sqrt{3})$ So $\mathrm{Re}(z^{2018})=\frac{1}{2}$ and $\mathrm{Im}(z^{2018})=-\frac{\sqrt{3}}{2}$.	
Mathematical behaviours	**Marks**
• derives the result that $z^{12}=1$ • observes that $z^{2018}=z^2$ • deduces the correct values of the real and imaginary parts	1 1 1

Question 6 (a) **(5 marks)**

Solution
Consider the augmented matrix $\left(\begin{array}{ccc\|c} 1 & -2 & 1 & 7 \\ 2 & 1 & -2 & 1 \\ -1 & \alpha & 2 & \beta \end{array}\right) \Rightarrow \left(\begin{array}{ccc\|c} 1 & -2 & 1 & 7 \\ 0 & 5 & -4 & -13 \\ 0 & \alpha-2 & 3 & \beta+7 \end{array}\right) \Rightarrow \left(\begin{array}{ccc\|c} 1 & -2 & 1 & 7 \\ 0 & 5 & -4 & -13 \\ 0 & \alpha+\frac{7}{4} & 0 & \beta-\frac{11}{4} \end{array}\right)$ Thus if $\alpha \neq -7/4$ the system will have a unique solution. If $\alpha = -7/4$ then the third equation is inconsistent unless $\beta = 11/4$. Hence no solution possible if $\alpha = -7/4$ and $\beta \neq 11/4$.

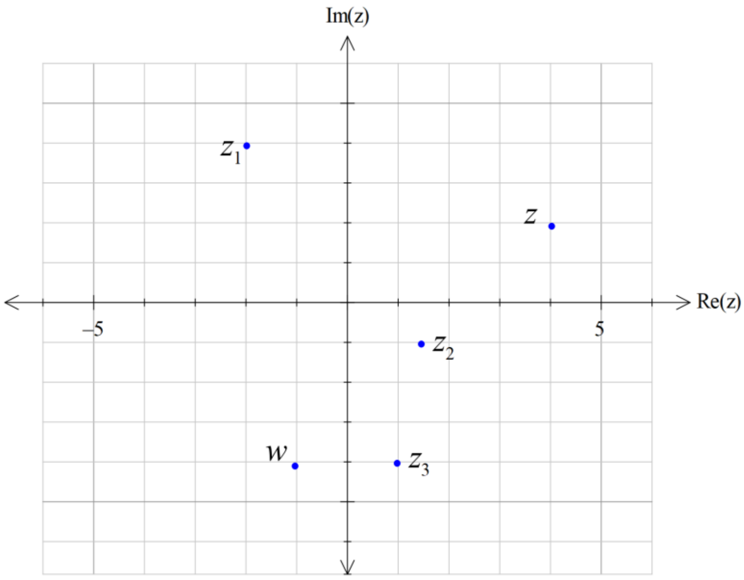
Mathematical behaviours	Marks
• sets up the augmented matrix • performs correctly one relevant row operation • performs correctly a second relevant row operation • notes that α has to have a particular value for no solution • notes that in addition β must be unequal to another critical value	1 1 1 1 1

Question 6 (b) **(2 marks)**

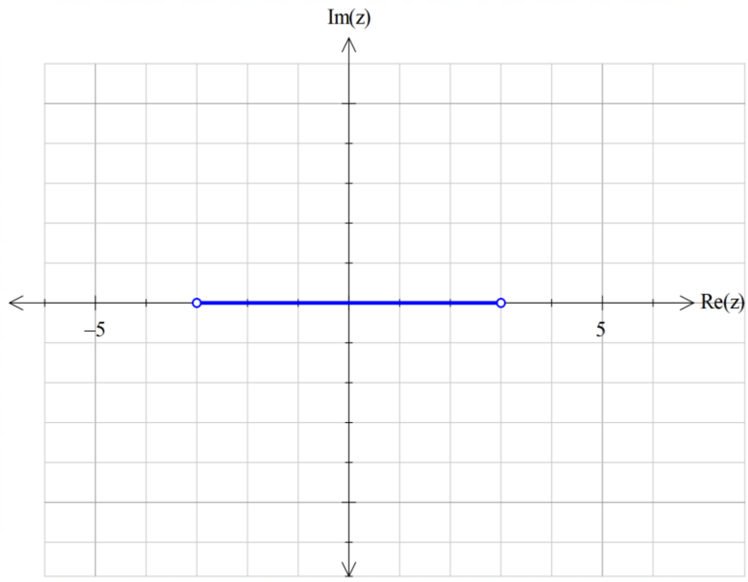
Solution
If $(x, y, z) = (3, -1, 2)$ is to be a unique solution the third equation forces $-\alpha + 1 = \beta \quad \Rightarrow \quad \alpha + \beta = 1$ Moreover we still need $\alpha \neq -7/4$ else the systems admits an infinity of solutions

Mathematical behaviours	Marks
• substitutes the given solution into the last equation to determine the connection between α and β • adds the restriction on α for otherwise the solution is not unique	1 1

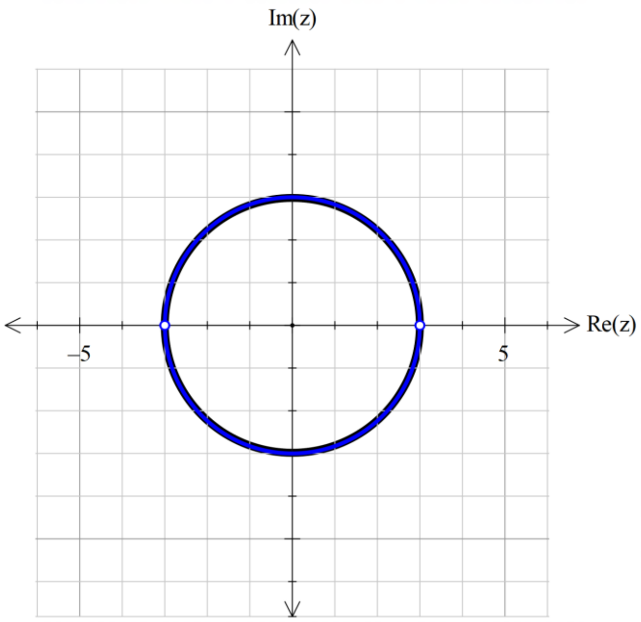
Question 7 (a) **(3 marks)**

Solution	
	
Mathematical behaviours	**Marks**
• plots correct location for z_1 • plots correct location for z_2 • plots correct location for z_3	1 1 1

Question 7 (b) **(2 marks)**

Solution	
	
Mathematical behaviours	Marks
• shows a line segment	1
• plots the correct end-points and -3 and 3 (technically these end points are not in the domain as the argument of 0 is not defined – but no marks need be deducted for not discussing this)	1

Question 7 (c) **(2 marks)**

Solution	
	
Mathematical behaviours	Marks
• plots a circular shape • shows the correct centre at the origin and the radius 3 (again, technically, the points 3 and -3 are not in the domain)	1 1